MAHESH PUBLIC SCHOOL, JODHPUR

STUDY MATERIAL

CLASS XI ***SUBJECT:-MATHEMATICS*** ***CHAPTER:TRIGONOMETRY***

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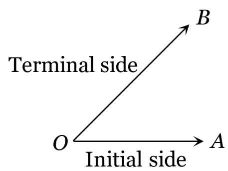
The word trigonometry is derived from two greek words 'trigonon' and 'metron'. The word 'trigonon' means a triangle and the word 'metron' means a measure. Hence the word trigonometry means the study of properties of triangles. This involves the measurement of angles and lengths.



The motion of any revolving line in a plane from its initial position (initial side) to the final position (terminal side) is called angle. The end point *O* about which the line rotates is called the vertex of the angle.

System of measurement of angles

There are three system for measuring angles

(1) **Sexagesimal or English system :** Therefore,

1 right angle = 90 degree $(=90^o)$

$1^o = 60$ minutes $(=60')$

$1' = 60$ second $(=60'')$

(2) **Centesimal or French system :** Therefore,

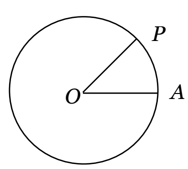
1 right angle = 100 grades $(=100^g)$

1 grade = 100 minutes $(=100')$

1 minute = 100 seconds $(=100'')$

(3) **Circular system :** The measure of an angle subtended at the centre of a circle by an arc of length equal to the radius of the circle.

Consider a circle of radius *r* having centre at *O*. Let *A* be a point on the circle. Now cut off an arc *AP* whose length is equal to the radius *r* of the circle. Then by the definition the measure of $\angle AOP$ is 1 radian $(=1^c)$.



Relation between three systems of measurement of an angle

Let D be the number of degrees, R be the number of radians and G be the number of grades in an angle θ, then

$$\frac{D}{90}=\frac{G}{100}=\frac{2R}{\pi}$$

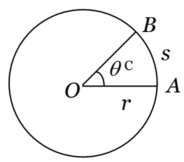
This is the required relation between the three systems of measurement of an angle.

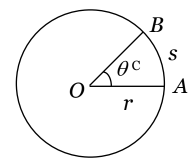
Therefore, one radian $=\frac{180^o}{\pi}\Rightarrow \pi$ radians $=180^o$

i.e., 1 radian = $57°17'44.8'' \approx 57°17'45''$.

Relation between an arc and an angle

If s is the length of an arc of a circle of radius r, then the angle θ (in radians) subtended by this arc at the centre of the circle is given by $\theta=\frac{s}{r}$ or $s=r\theta$.





i.e., Arc = radius × angle in radians

Sectorial area : Let OAB be a sector having central angle θ^C and radius r. Then area of the sector OAB is given by $\frac{1}{2}r^2\theta$.

Domain and range of a trigonometrical function

If $f: X \to Y$ is a function, defined on the set X, then the **domain** of the function f, written as Domf is the set of all independent variables x, for which the image $f(x)$ is well defined element of Y, called the co-domain of f.

Range of $f: X \to Y$ is the set of all images $f(x)$ which belongs to Y, *i.e.,* Range $f = \{f(x)\in Y : x\in X\}\subseteq Y$.

The domain and range of trigonometrical functions are tabulated as follows :

Table : 10.1

Trigonometrical Function	Domain	Range
$\sin x$	R	$-1\le \sin x\le 1$
$\cos x$	R	$-1\le \cos x\le 1$
$\tan x$	$R-\left\{(2n+1)\frac{\pi}{2}, n\in I\right\}$	R
$\text{cosec } x$	$R-\{n\pi, n\in I\}$	$R-\{x:-1<x<1\}$
$\sec x$	$R-\left\{(2n+1)\frac{\pi}{2}, n\in I\right\}$	$R-\{x:-1<x<1\}$
$\cot x$	$R-\{n\pi, n\in I\}$	R

Trigonometrical ratios or functions

In the right angled triangle OMP, we have base = $OM = x$, perpendicular $= PM = y$ and hypotenues = $OP = r$. We define the following trigonometric ratio which are also known as trigonometric function.

$$\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenues}} = \frac{y}{r}$$

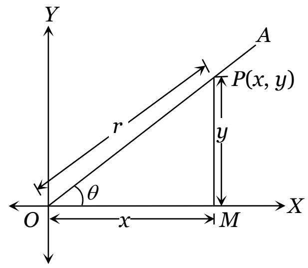
$$\cos\theta = \frac{\text{Base}}{\text{Hypotenues}} = \frac{x}{r}$$

$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{y}{x}$$

$$\cot\theta = \frac{\text{Base}}{\text{Perpendicular}} = \frac{x}{y}$$

$$\sec\theta = \frac{\text{Hypotenues}}{\text{Base}} = \frac{r}{x}$$

$$\text{cosec}\,\theta = \frac{\text{Hypotenues}}{\text{Perpendicular}} = \frac{r}{y}$$



(1) **Relation between trigonometric ratios (functions)**

(i) $\sin\theta.\text{cosec}\theta = 1$ (ii) $\tan\theta.\cot\theta = 1$

(iii) $\cos\theta.\sec\theta = 1$ (iv) $\tan\theta = \frac{\sin\theta}{\cos\theta}$ (v) $\cot\theta = \frac{\cos\theta}{\sin\theta}$

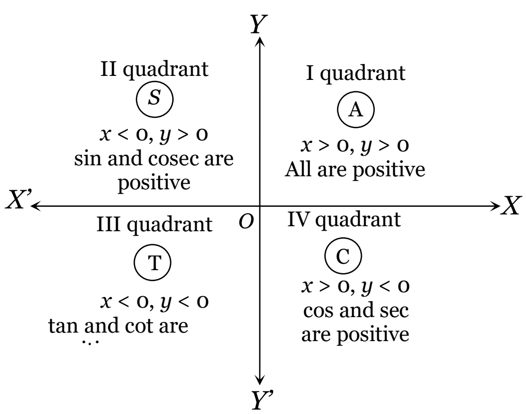
(2) **Fundamental trigonometric identities**

(i) $\sin^2\theta + \cos^2\theta = 1$ (ii) $1 + \tan^2\theta = \sec^2\theta$

(iii) $1 + \cot^2\theta = \text{cosec}^2\theta$

(3) **Sign of trigonometrical ratios or functions :** Their signs depends on the quadrant in which the terminal side of the angle lies.

In brief: A crude aid to memorise the signs of trigonometrical ratio in different quadrant. "**Add Sugar To Coffee**".

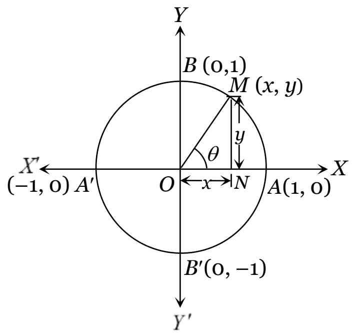


Algorithm : First determine the sign of the trigonometric function.

If θ is measured from $X'OX$ *i.e.*, $\{(\pi \pm \theta, 2\pi - \theta)\}$ then retain the original name of the function.

If θ is measured from $Y'OY$ *i.e.*, $\left\{\frac{\pi}{2}\pm\theta, \frac{3\pi}{2}\pm\theta\right\}$, then change sine to cosine, cosine to sine, tangent to cotangent, cot to tan, sec to cosec and cosec to sec.

(4) **Variations in values of trigonometric functions in different quadrants :** Let $X'OX$ and YOY' be the coordinate axes. Draw a circle with centre at origin O and radius unity.



Let $M(x,y)$ be a point on the circle such that $\angle AOM=\theta$ then $x=\cos\theta$ and $y=\sin\theta$; $-1\le\cos\theta\le 1$ and $-1\le\sin\theta\le 1$ for all values of θ.

Table : 10.2

II-Quadrant (S)	**I-Quadrant (A)**
$\sin\theta\to$ decreases from 1 to 0	$\sin\theta\to$ increases from 0 to 1
$\cos\theta\to$ decreases from 0 to – 1	$\cos\theta\to$ decreases from 1 to 0
$\tan\theta\to$ increases from – ∞ to 0	$\tan\theta\to$ increases from 0 to ∞
$\cot\theta\to$ decreases from 0 to – ∞	$\cot\theta\to$ decreases from ∞ to 0
$\sec\theta\to$ increases from – ∞ to – 1	$\sec\theta\to$ increases from 1 to ∞
$\text{cosec}\,\theta\to$ increases from 1 to ∞	$\text{cosec}\,\theta\to$ decreases from ∞ to 1
III-Quadrant (T)	**IV-Quadrant (C)**
$\sin\theta\to$ decreases from 0 to – 1	$\sin\theta\to$ increases from – 1 to 0
$\cos\theta\to$ increases from – 1 to 0	$\cos\theta\to$ increases from 0 to 1
$\tan\theta\to$ increases from 0 to ∞	$\tan\theta\to$ increases from – ∞ to 0
$\cot\theta\to$ decreases from ∞ to 0	$\cot\theta\to$ decreases from 0 to – ∞
$\sec\theta\to$ decreases from – 1 to – ∞	$\sec\theta\to$ decreases from ∞ to 1
$\text{cosec}\,\theta\to$ increases from – ∞ to – 1	$\text{cosec}\,\theta\to$ decreases from –1 to –∞

Trigonometrical ratios of allied angles

Two angles are said to be allied when their sum or difference is either zero or a multiple of 90^o.

Table : 10.3

Allied angles → Trigo. Ratio	$\sin\theta$	$\cos\theta$	$\tan\theta$
$(-\theta)$	$-\sin\theta$	$\cos\theta$	$-\tan\theta$
$(90-\theta)$ or $\left(\frac{\pi}{2}-\theta\right)$	$\cos\theta$	$\sin\theta$	$\cot\theta$
$(90-\theta)$ or $\left(\frac{\pi}{2}-\theta\right)$	$\cos\theta$	$-\sin\theta$	$-\cot\theta$
$(180-\theta)$ or $(\pi-\theta)$	$\sin\theta$	$-\cos\theta$	$-\tan\theta$
$(180+\theta)$ or $(\pi+\theta)$	$-\sin\theta$	$-\cos\theta$	$\tan\theta$
$(270-\theta)$ or $\left(\frac{3\pi}{2}-\theta\right)$	$-\cos\theta$	$-\sin\theta$	$\cot\theta$
$(270+\theta)$ or $\left(\frac{3\pi}{2}+\theta\right)$	$-\cos\theta$	$\sin\theta$	$-\cot\theta$
$(360-\theta)$ or $(2\pi-\theta)$	$-\sin\theta$	$\cos\theta$	$-\tan\theta$

Trigonometrical ratios for various angles

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	π	$3\pi/2$	2π
$\sin\theta$	0	1/2	$1/\sqrt{2}$	$\sqrt{3}/2$	1	0	−1	0
$\cos\theta$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	1/2	0	−1	0	1
$\tan\theta$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	∞	0	∞	0

Trigonometrical ratios for some special angles

θ	$7\frac{1}{2}^o$	15^o	$22\frac{1}{2}^o$	18^o	36^o

$\sin\theta$	$\frac{\sqrt{4-\sqrt{2}-\sqrt{6}}}{2\sqrt{2}}$	$\frac{\sqrt{3}-1}{2\sqrt{2}}$	$\frac{1}{2}\sqrt{2-\sqrt{2}}$	$\frac{\sqrt{5}-1}{4}$	$\frac{1}{4}\sqrt{10-2\sqrt{5}}$
$\cos\theta$	$\frac{\sqrt{4+\sqrt{2}+\sqrt{6}}}{2\sqrt{2}}$	$\frac{\sqrt{3}+1}{2\sqrt{2}}$	$\frac{1}{2}\sqrt{2+\sqrt{2}}$	$\frac{1}{4}\sqrt{10+2\sqrt{5}}$	$\frac{\sqrt{5}+1}{4}$
$\tan\theta$	$(\sqrt{3}-\sqrt{2})(\sqrt{2}-1)$	$2-\sqrt{3}$	$\sqrt{2}-1$	$\frac{\sqrt{25-10\sqrt{15}}}{5}$	$\sqrt{5-2\sqrt{5}}$

Trigonometrical ratios in terms of each other

	sin $\boldsymbol{\theta}$	**cos** $\boldsymbol{\theta}$	**tan** $\boldsymbol{\theta}$	**cot**$\boldsymbol{\theta}$	**sec**$\boldsymbol{\theta}$	**cosec**$\boldsymbol{\theta}$
sin $\boldsymbol{\theta}$	$\sin\theta$	$\sqrt{1-\cos^2\theta}$	$\frac{\tan\theta}{\sqrt{1+\tan^2\theta}}$	$\frac{1}{\sqrt{1+\cot^2\theta}}$	$\frac{\sqrt{\sec^2\theta-1}}{\sec\theta}$	$\frac{1}{\operatorname{cosec}\theta}$
cos $\boldsymbol{\theta}$	$\sqrt{1-\sin^2\theta}$	$\cos\theta$	$\frac{1}{\sqrt{1+\tan^2\theta}}$	$\frac{\cot\theta}{\sqrt{1+\cot^2\theta}}$	$\frac{1}{\sec\theta}$	$\frac{\sqrt{\operatorname{cosec}^2\theta-1}}{\operatorname{cosec}\theta}$
tan $\boldsymbol{\theta}$	$\frac{\sin\theta}{\sqrt{1-\sin^2\theta}}$	$\frac{\sqrt{1-\cos^2\theta}}{\cos\theta}$	$\tan\theta$	$\frac{1}{\cot\theta}$	$\sqrt{\sec^2\theta-1}$	$\frac{1}{\sqrt{\operatorname{cosec}^2\theta-1}}$
cot $\boldsymbol{\theta}$	$\frac{\sqrt{1-\sin^2\theta}}{\sin\theta}$	$\frac{\cos\theta}{\sqrt{1-\cos^2\theta}}$	$\frac{1}{\tan\theta}$	$\cot\theta$	$\frac{1}{\sqrt{\sec^2\theta-1}}$	$\sqrt{\operatorname{cosec}^2\theta-1}$
sec $\boldsymbol{\theta}$	$\frac{1}{\sqrt{1-\sin^2\theta}}$	$\frac{1}{\cos\theta}$	$\sqrt{1+\tan^2\theta}$	$\frac{\sqrt{1+\cot^2\theta}}{\cot\theta}$	$\sec\theta$	$\frac{\operatorname{cosec}\theta}{\sqrt{\operatorname{cosec}^2\theta-1}}$
cosec$\boldsymbol{\theta}$	$\frac{1}{\sin\theta}$	$\frac{1}{\sqrt{1-\cos^2\theta}}$	$\frac{\sqrt{1+\tan^2\theta}}{\tan\theta}$	$\sqrt{1+\cot^2\theta}$	$\frac{\sec\theta}{\sqrt{\sec^2\theta-1}}$	$\operatorname{cosec}\theta$

Formulae for the trigonometric ratios of sum and differences of two angles

(1) $\sin(A+B)=\sin A\cos B+\cos A\sin B$

(2) $\sin(A-B)=\sin A\cos B-\cos A\sin B$

(3) $\cos(A+B)=\cos A\cos B-\sin A\sin B$

(4) $\cos(A-B)=\cos A\cos B+\sin A\sin B$

(5) $\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$

(6) $\tan(A-B)=\frac{\tan A-\tan B}{1+\tan A\tan B}$

(7) $\cot(A+B)=\frac{\cot A\cot B-1}{\cot A+\cot B}$

(8) $\cot(A-B)=\frac{\cot A\cot B+1}{\cot B-\cot A}$

(9) $\sin(A+B)\sin(A-B)=\sin^2 A-\sin^2 B=\cos^2 B-\cos^2 A$

(10) $\cos(A+B)\cos(A-B)=\cos^2 A-\sin^2 B=\cos^2 B-\sin^2 A$

(11) $\tan A\pm\tan B=\frac{\sin A}{\cos A}\pm\frac{\sin B}{\cos B}=\frac{\sin A\cos B\pm\cos A\sin B}{\cos A\cos B}$

$$= \frac{\sin(A \pm B)}{\cos A . \cos B}, \quad \left(A \neq n\pi + \frac{\pi}{2}, B \neq m\pi\right)$$

(12) $\cot A \pm \cot B = \frac{\sin(B \pm A)}{\sin A \sin B}, \quad \left(A \neq n\pi, B \neq m\pi + \frac{\pi}{2}\right)$

Formulae for the trigonometric ratios of sum and differences of three angles

(1) $\sin(A+B+C) = \sin A \cos B \cos C + \cos A \sin B \cos C$

$$+ \cos A \cos B \sin C - \sin A \sin B \sin C$$

or sin $(A+B+C) = \cos A \cos B \cos C(\tan A + \tan B$

$$+ \tan C - \tan A . \tan B . \tan C)$$

(2) $\cos(A+B+C) = \cos A \cos B \cos C - \sin A \sin B \cos C$

$$- \sin A \cos B \sin C - \cos A \sin B \sin C$$

$\cos(A+B+C) =$

$$\cos A \cos B \cos C(1 - \tan A \tan B - \tan B \tan C - \tan C \tan A)$$

(3) $\tan(A+B+C) = \frac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A \tan B - \tan B \tan C - \tan C \tan A}$

(4) $\cot(A+B+C) = \frac{\cot A \cot B \cot C - \cot A - \cot B - \cot C}{\cot A \cot B + \cot B \cot C + \cot C . \cot A - 1}$

In general :

(5) $\sin(A_1 + A_2 + \ldots\ldots + A_n)$

$$= \cos A_1 \cos A_2 \ldots\ldots \cos A_n (S_1 - S_3 + S_5 - S_7 + \ldots)$$

(6) $\cos(A_1 + A_2 + .. + A_n) = \cos A_1 \cos A_2 .. \cos A_n (1 - S_2 + S_4 - S_6 ..)$

(7) $\tan(A_1 + A_2 + \ldots.. + A_n) = \frac{S_1 - S_3 + S_5 - S_7 + \ldots.}{1 - S_2 + S_4 - S_6 + \ldots.}$

where, $S_1 = \tan A_1 + \tan A_2 + \ldots. + \tan A_n$ = The sum of the tangents of the separate angles.

$S_2 = \tan A_1 \tan A_2 + \tan A_1 \tan A_3 + \ldots.$ = The sum of the tangents taken two at a time.

$S_3 = \tan A_1 \tan A_2 \tan A_3 + \tan A_2 \tan A_3 \tan A_4 + \ldots$ = Sum of tangents three at a time, and so on.

If $A_1 = A_2 = \ldots. = A_n = A,$ then $S_1 = n \tan A$,

$S_2 = {}^nC_2 \tan^2 A$, $S_3 = {}^nC_3 \tan^3 A, \ldots.$

(8) $\sin nA = \cos^n A({}^nC_1 \tan A - {}^nC_3 \tan^3 A + {}^nC_5 \tan^5 A - \ldots.)$

(9) $\cos nA = \cos^n A(1 - {}^nC_2 \tan^2 A + {}^nC_4 \tan^4 A - \ldots)$

(10) $\tan nA = \frac{{}^nC_1 \tan A - {}^nC_3 \tan^3 A + {}^nC_5 \tan^5 A - \ldots.}{1 - {}^nC_2 \tan^2 A + {}^nC_4 \tan^4 A - {}^nC_6 \tan^6 A + \ldots}$

(11) $\sin nA + \cos nA = \cos^n A(1 + {}^nC_1 \tan A - {}^nC_2 \tan^2 A$

$$- {}^nC_3 \tan^3 A + {}^nC_4 \tan^4 A + {}^nC_5 \tan^5 A - {}^nC_6 \tan^6 A - \ldots..)$$

(12) $\sin nA - \cos nA = \cos^n A(-1 + {}^nC_1 \tan A)$

$+ {}^nC_2 \tan^2 A - {}^nC_3 \tan^3 A - {}^nC_4 \tan^4 A + {}^nC_5 \tan^5 A + {}^nC_6 \tan^6 A + \ldots..)$

(13) $\sin(\alpha) + \sin(\alpha + \beta) + \sin(\alpha + 2\beta) + \ldots.. + \sin(\alpha + (n-1)\beta)$

$$= \frac{\sin\{\alpha + (n-1)(\beta/2)\} . \sin(n\beta/2)}{\sin(\beta/2)}$$

(14) $\cos(\alpha) + \cos(\alpha + \beta) + \cos(\alpha + 2\beta) + \ldots. + \cos(\alpha + (n-1)\beta)$

$$= \frac{\cos\left\{\alpha + (n-1)\left(\frac{\beta}{2}\right)\right\}.\sin\left\{n\left(\frac{\beta}{2}\right)\right\}}{\sin\left(\frac{\beta}{2}\right)}$$

Formulae to transform the product into sum or difference

(1) $2\sin A\cos B = \sin(A+B) + \sin(A-B)$

(2) $2\cos A\sin B = \sin(A+B) - \sin(A-B)$

(3) $2\cos A\cos B = \cos(A+B) + \cos(A-B)$

(4) $2\sin A\sin B = \cos(A-B) - \cos(A+B)$

Let $A+B=C$ and $A-B=D$

Then, $A = \frac{C+D}{2}$ and $B = \frac{C-D}{2}$

Therefore, we find out the formulae to transform the sum or difference into product.

(1) $\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$

(2) $\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$

(3) $\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$

(4) $\cos C - \cos D = 2\sin\frac{C+D}{2}\sin\frac{D-C}{2} = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}$.

Trigonometric ratio of multiple of an angle

(1) $\sin 2A = 2\sin A\cos A = \frac{2\tan A}{1+\tan^2 A}$

(2) $\cos 2A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$

$= \cos^2 A - \sin^2 A = \frac{1-\tan^2 A}{1+\tan^2 A}$; where $A \neq (2n+1)\frac{\pi}{4}$.

(3) $\tan 2A = \frac{2\tan A}{1-\tan^2 A}$

(4) $\sin 3A = 3\sin A - 4\sin^3 A$

$= 4\sin(60^o - A).\sin A.\sin(60^o + A)$

(5) $\cos 3A = 4\cos^3 A - 3\cos A$

$= 4\cos(60^o - A).\cos A.\cos(60^o + A)$

(6) $\tan 3A = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A} = \tan(60^o - A).\tan A.\tan(60^o + A)$,

where $A \neq n\pi + \pi/6$

(7) $\sin 4\theta = 4\sin\theta.\cos^3\theta - 4\cos\theta\sin^3\theta$

(8) $\cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1$

(9) $\tan 4\theta = \frac{4\tan\theta - 4\tan^3\theta}{1 - 6\tan^2\theta + \tan^4\theta}$

(10) $\sin 5A = 16\sin^5 A - 20\sin^3 A + 5\sin A$

(11) $\cos 5A = 16\cos^5 A - 20\cos^3 A + 5\cos A$

Trigonometric ratio of sub-multiple of an angle

(1) $\left|\sin\frac{A}{2}+\cos\frac{A}{2}\right| = \sqrt{1+\sin A}$

or $\sin\frac{A}{2}+\cos\frac{A}{2} = \pm\sqrt{1+\sin A}$

i.e., $\begin{cases} +, \text{If } 2n\pi - \pi/4 \le A/2 \le 2n\pi + \frac{3\pi}{4} \\ -, \text{otherwise} \end{cases}$

(2) $\left|\sin\frac{A}{2}-\cos\frac{A}{2}\right| = \sqrt{1-\sin A}$

or $(\sin\frac{A}{2}-\cos\frac{A}{2}) = \pm\sqrt{1-\sin A}$

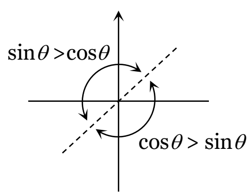
i.e., $\begin{cases} +, \text{If } 2n\pi + \pi/4 \le A/2 \le 2n\pi + \frac{5\pi}{4} \\ -, \text{otherwise} \end{cases}$

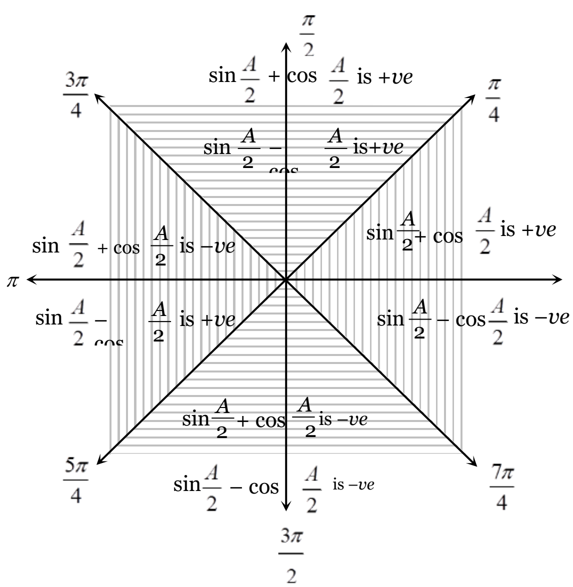
(3) (i) $\tan\frac{A}{2} = \frac{\pm\sqrt{\tan^2 A+1}-1}{\tan A} = \pm\sqrt{\frac{1-\cos A}{1+\cos A}} = \frac{1-\cos A}{\sin A}$,

where $A \ne (2n+1)\pi$

(ii) $\cot\frac{A}{2} = \pm\sqrt{\frac{1+\cos A}{1-\cos A}} = \frac{1+\cos A}{\sin A}$, where $A \ne 2n\pi$

The ambiguities of signs are removed by locating the quadrants in which $\frac{A}{2}$ lies or you can follow the following figure,





(4) $\tan^2 \frac{A}{2} = \frac{1 - \cos A}{1 + \cos A}$; where $A \neq (2n+1)\pi$

(5) $\cot^2 \frac{A}{2} = \frac{1 + \cos A}{1 - \cos A}$; where $A \neq 2n\pi$

Maximum and minimum value of *a* cosθ + *b* sinθ

Let $a = r\cos\alpha$ .....(i) and $b = r\sin\alpha$ .....(ii)

Squaring and adding (i) and (ii), then $a^2 + b^2 = r^2$ or, $r = \sqrt{a^2 + b^2}$

$\therefore$ $a\sin\theta + b\cos\theta = r(\sin\theta\cos\alpha + \cos\theta\sin\alpha) = r\sin(\theta+\alpha)$

But $-1 \le \sin\theta < 1$ So, $-1 \le \sin(\theta+\alpha) \le 1$;

Then $-r \le r\sin(\theta+\alpha) \le r$

Hence, $-\sqrt{a^2+b^2} \le a\sin\theta + b\cos\theta \le \sqrt{a^2+b^2}$

Then the greatest and least values of $a\sin\theta + b\cos\theta$ are respectively $\sqrt{a^2+b^2}$ and $-\sqrt{a^2+b^2}$

.

Therefore, $\sin^2 x + \operatorname{cosec}^2 x \ge 2,$ for every real x.

$\cos^2 x + \sec^2 x \ge 2,$ for every real x.

$\tan^2 x + \cot^2 x \ge 2$, for every real x.

Conditional trigonometrical identities

We have certain trigonometric identities.

Like, $\sin^2\theta + \cos^2\theta = 1$ and $1 + \tan^2\theta = \sec^2\theta$ etc.

Such identities are identities in the sense that they hold for all value of the angles which satisfy the given condition among them and they are called conditional identities.

(1) If $A + B + C = 180^o$, then

(i) $\sin 2A + \sin 2B + \sin 2C = 4\sin A\sin B\sin C$

(ii) $\sin 2A + \sin 2B - \sin 2C = 4\cos A\cos B\sin C$

(iii) $\sin(B + C - A) + \sin(C + A - B) + \sin(A + B - C)$

$= 4\sin A\sin B\sin C$

(iv) $\cos 2A + \cos 2B + \cos 2C = -1 - 4\cos A\cos B\cos C$

(v) $\cos 2A + \cos 2B - \cos 2C = 1 - 4\sin A\sin B\cos C$

(2) If $A + B + C = 180^o$, then

(i) $\sin A + \sin B + \sin C = 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$

(ii) $\sin A + \sin B - \sin C = 4\sin\frac{A}{2}\sin\frac{B}{2}\cos\frac{C}{2}$

(iii) $\cos A + \cos B + \cos C = 1 + 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

(iv) $\cos A + \cos B - \cos C = -1 + 4\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$

(v) $\frac{\cos A}{\sin B\sin C} + \frac{\cos B}{\sin C\sin A} + \frac{\cos C}{\sin A\sin B} = 2$

(3) If $A + B + C = \pi$, then

(i) $\sin^2 A + \sin^2 B - \sin^2 C = 2\sin A\sin B\cos C$

(ii) $\cos^2 A+\cos^2 B+\cos^2 C=1-2\cos A\cos B\cos C$

(iii) $\sin^2 A+\sin^2 B+\sin^2 C=1-2\sin A\sin B\cos C$

(4) If $A+B+C=\pi,$ then

(i) $\sin^2\frac{A}{2}+\sin^2\frac{B}{2}+\sin^2\frac{C}{2}=1-2\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

(ii) $\cos^2\frac{A}{2}+\cos^2\frac{B}{2}+\cos^2\frac{C}{2}=2+2\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

(iii) $\sin^2\frac{A}{2}+\sin^2\frac{B}{2}-\sin^2\frac{C}{2}=1-2\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$

(iv) $\cos^2\frac{A}{2}+\cos^2\frac{B}{2}-\cos^2\frac{C}{2}=2\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$

(5) If $x+y+z=\frac{\pi}{2}$, then

(i) $\sin^2 x+\sin^2 y+\sin^2 z=1-2\sin x\sin y\sin z$

(ii) $\cos^2 x+\cos^2 y+\cos^2 z=2+2\sin x\sin y\sin z$

(iii) $\sin 2x+\sin 2y+\sin 2z=4\cos x\cos y\cos z$

(6) If $A+B+C=\pi$, then

(i) $\tan A+\tan B+\tan C=\tan A\tan B\tan C$

(ii) $\cot B\cot C+\cot C\cot A+\cot A\cot B=1$

(iii) $\tan\frac{B}{2}\tan\frac{C}{2}+\tan\frac{C}{2}\tan\frac{A}{2}+\tan\frac{A}{2}\tan\frac{B}{2}=1$

(iv) $\cot\frac{A}{2}+\cot\frac{B}{2}+\cot\frac{C}{2}=\cot\frac{A}{2}\cot\frac{B}{2}\cot\frac{C}{2}$