Notes

Chapter-Rational Numbers

Class VIII

Natural numbers:

The groups of the positive numbers which are countable are known as natural numbers. The examples of natural numbers will be 1, 2, 3, 4, 5, etc.
On solving equations like $x + 3 = 12$, we get the solution as $x = 9$. The solution 9 is a natural number.

Whole numbers:

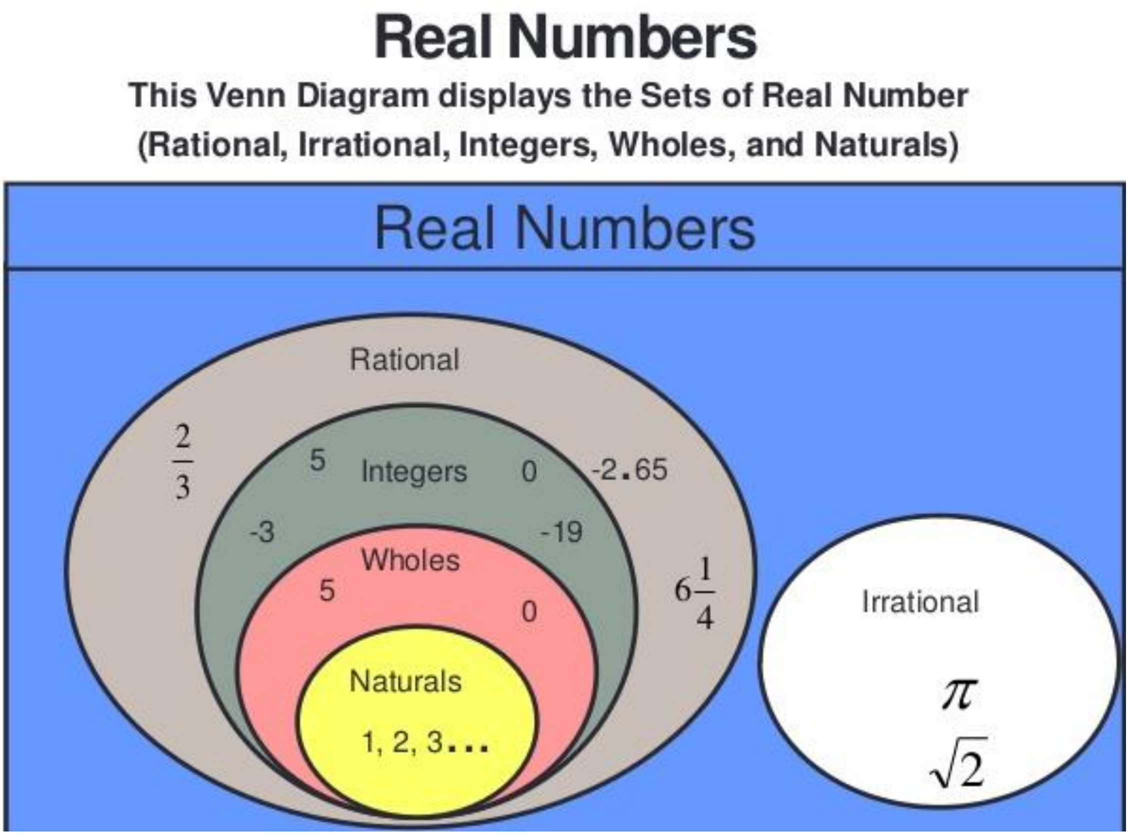
The groups of natural numbers with inclusion of zero in it are known as whole numbers. The examples of whole numbers will be 0, 1, 2, 3, 4, 5, etc.
On solving equations like $x + 7 = 7$, we get the solution as $x = 0$. The solution 0 is a whole number.
For the given equation, if we would have considered only the natural numbers, then we could not have found the solution. By adding zero to group of natural numbers we get the whole numbers.

Integers:

The groups of positive and negative numbers along with zero are known as integer numbers. The examples of integer numbers will be -3, -2, -1, 0, 1, 2, 3, 4, 5, etc.
On solving equations like $x + 12 = 7$, we get the solution as $x = -5$. The solution -5 is an integer.
For the given equation, if we would have considered only the whole numbers, then we could not have found the solution. By adding negative numbers to group of whole numbers we get the integer numbers.

Rational numbers:

The numbers which can be expressed as ratio of integers are known as rational numbers. The examples of rational numbers will be 1/4, 2/7, - 3/10, 34/7, etc.
On solving equations like $3x + 5 = 0$, we get the solution as $x = -5/3$. The solution -5/3 is neither a natural number or whole number or integer.
This leads us to the collection of Rational Numbers. These are the numbers which can be expressed in x/y form; where $y \neq 0$.



Properties of Rational Numbers:

(i) Closure Property:

When any operation is performed between two or more rational numbers and their result is also a rational number then we say that the rational numbers follow the closure property for that operation.

Operation	Numbers	Remark
Addition	a) 5/3 + 3/2 = 19/6 (Rational No); b) 7/3 +(– 5/2) = -1/6 (Rational No); ……	We can observe that addition of two rational numbers x and y, i.e. x + y is always a rational number. Hence, rational numbers are closed under addition.
Subtraction	a) 5/3 - 3/2 = 1/6 (Rational No);	We can observe that subtraction of two rational numbers x and y, i.e. x - y is

	b) -7/3 – 5/2 = -29/6 (Rational No); ……	always a rational number. Hence, rational numbers are closed under subtraction.
Multiplication	a) 5/ 3 × 3/2 = 5/2 (Rational No); b) -2/7 × 14/5 = -4/5 (Rational No); ……	We can observe that multiplication of two rational numbers x and y, i.e. x × y is always a rational number. Hence, rational numbers are closed under multiplication.
Division	a) 5/3 ÷ 3/2 = 10/9 (Rational No); b) 12/3 ÷ 0 = ∞ (Not a rational no); …..	We can observe that division of two rational numbers x and y, i.e. x ÷ y is not always a rational number. Hence, rational numbers are not closed under division.

(ii) Commutative Property:
When two rational numbers are swapped between one operator and still their result does not change then we say that the rational numbers follow the commutative property for that operation.

Operation	**Numbers**	**Remark**
Addition	(a) 5/3 + 3/2 = 19/6; 3/2 + 5/3 = 19/6 Here, both answers are same (b) 7/3 + (-5/2) = -1/6; (-5/2) + 7/3 = -1/6 Here, both answers are same	We can observe that addition of two rational numbers x and y when inter changed yields the same answer, i.e. x + y = y + x.

	……	Hence, rational numbers are commutative under addition.
Subtraction	(a) 5/3 - 3/2 = 1/6; 3/2 – 5/3 = -1/6 Here, both answers are different(b) 7/3 – 5/2 = -1/6; 5/2 – 7/3 = 1/6 Here, both answers are different ……	We can observe that subtraction of two rational numbers x and y when inter changed does not yield the same answer, i.e. x - y ≠ y - x. Hence, rational numbers are not commutative under subtraction.
Multiplication	(a) 5/3 × 3/2 = 5/2; 3/2 × 5/3 = 5/2 Here, both answers are same (b) -2/7 × 14/5 = -4/5; 14/5 × (-2/7) = -4/5 Here, both answers are same ……	We can observe that multiplication of two rational numbers x and y when inter changed yields the same answer, i.e. x × y = y × x. Hence, rational numbers are commutative under multiplication.
Division	(a) 5/3 ÷ 3/2 = 10/9; 3/2 ÷ 5/3 = 9/10 Here, both answers are different(b) 12/3 ÷ 0 = ∞ ; 0 ÷ 12/3 = 0 ; Here, both answers are different …..	We can observe that division of two rational numbers x and y when inter changed does not yield the same answer, i.e. x ÷ y ≠ y ÷ x. Hence, rational numbers are not commutative under division.s

(iii) Associative Property:

When rational numbers are rearranged among two or more same operations and still their result does not change then we say that the rational numbers follow the associative property for that operation.

Operation	**Numbers**	**Remark**

Addition	(a) 5/3 + (3/2 + 1/3) = 7/2; (5/3 + 3/2) + 1/3 = 7/2 Here, both answers are same (b) 7/3 + (-5/2 + 1/4) = 1/12; (7/3 + -5/2) + 1/4 = 1/12; Here, both answers are same ……	We can observe that addition of rational numbers x, y, and z in any order yields the same answer, i.e. x + (y + z) = (x + y) + z. Hence, rational numbers are associative under addition.
Subtraction	(a) 5/3 – (3/2 – 1/3) = 3/2; (5/3 – 3/2) – 1/3 = -1/2 Here, both answers are different ……	We can observe that subtraction of rational numbers x, y, and z in any order does not yields the same answer, i.e. x - (y - z) ≠ (x - y) - z. Hence, rational numbers are not associative under subtraction.
Multiplication	(a) 5/ 3 × (3/2 × 2/3) = 5/3; (5/ 3 × 3/2) × 2/3 = 5/3 Here, both answers are same (b) -2/7 × (14/5 × 10/2) = -4; (-2/7 × 14/5) × 10/2 = -4 Here, both answers are same ……	We can observe that multiplication of rational numbers x, y, and z in any order yields the same answer, i.e. x × (y × z) = (x × y) × z. Hence, rational numbers are associative under multiplication.
Division	(a) 5/3 ÷ (3/2 ÷ 1/4) = 5/18; (5/3 ÷ 3/2) ÷ 1/4 = 40/9 Here, both answers are different …..	We can observe that division of rational numbers x, y, and z in any order does not yields the same answer, i.e. x ÷ (y ÷ z) ≠ (x ÷ y) ÷ z. Hence, rational numbers are not associative under division.

(iv) The role of zero (0):
When any rational number is added to zero (0), then it will result in the same rational number i.e. x/y + 0 = x/y.
Zero is called the identity for the addition of rational numbers.
***Example*:** 2/3 + 0 = 2/3; -5/7 + 0 = -5/7; etc.

(v) The role of one (1):
When any rational number is multiplied with one (1), then it will result in the same rational number i.e. x/y × 1 = x/y.
One is called the multiplicative identity for rational numbers.
***Example*:** 2/7 × 1 = 2/7; -8/3 × 1 = -8/3; etc.

(vi) **Negative of a number:**
When a rational number is added to the negative or additive inverse of its own, result will be zero (0) i.e. x/y + (-y/x) = 0.
***Example*:** 2/7 + (-7/2) = 0; etc.

(vii) Reciprocal of a number:
When a rational number is multiplied with the reciprocal or multiplicative inverse of its own, result will be one (1) i.e. x/y × y/x = 1.
***Example*:** 2/7 × 7/2 = 1; etc.

(viii) Distributive property of multiplication over addition:
If the rational numbers a, b, and c obey property of a × (b + c) = ab + ac, then it is said to follow Distributive property of multiplication over addition.
Example:
1/3 × (2/3 + 1/4) = 1/3 × 11/12 = 11/36………………….. (i)
(1/3 × 2/3) + (1/3 × 1/4) = 2/9 + 1/12 = 11/36……………. (ii)
Here, answer for both the equations (i) and (ii) are same.
Hence, rational numbers follow distributive property of multiplication over addition.

Some Examples:

***Example 1*:** $\frac{2}{5}\times\left(-\frac{3}{7}\right)-\frac{1}{6}\times\frac{3}{2}+\frac{1}{14}\times\frac{2}{5}$

Solution: $\frac{2}{5}\times\left(-\frac{3}{7}\right)-\frac{1}{6}\times\frac{3}{2}+\frac{1}{14}\times\frac{2}{5}$

$= \frac{2}{5} \times \left(-\frac{3}{7}\right) + \frac{1}{14} \times \frac{2}{5} - \frac{1}{6} \times \frac{3}{2}$ (Using commutative property)

On using distributive property,

$= \frac{2}{5}\left(-\frac{3}{7} + \frac{1}{14}\right) - \frac{1}{6} \times \frac{3}{2}$

$= \frac{2}{5}\left(\frac{-6+1}{14}\right) - \frac{1}{6} \times \frac{3}{2}$

$= \frac{2}{5}\left(\frac{-5}{14}\right) - \frac{1}{6} \times \frac{3}{2}$

$= \frac{2}{5} \times \frac{-5}{14} - \frac{1}{4}$

$= \frac{-1}{7} - \frac{1}{4}$

$= \frac{-4-7}{28} = \frac{-11}{28}$

***Example 2*:** Write the additive inverse of 19/-6.
Solution:We know,

$\frac{19}{-6} + \frac{19}{6} = \frac{-19+19}{6} = 0$

Hence, the additive inverse of $\frac{19}{-6} \, is \, \frac{19}{6}$

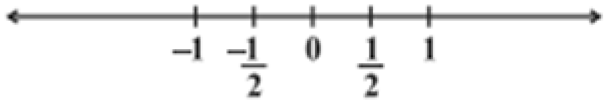
***Example 3*:** Write the multiplicative inverse of $-1 \times \frac{-2}{5}$

Solution: When rational numbers are swapped between one operators and still their result does not change, then we say that the numbers follow the commutative property for that operation.
Hence, commutative property is used here.

1. Representation of Rational Numbers on the Number Line:

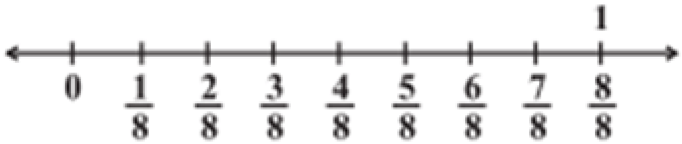
The number line for rational line will extend from - ∞ to ∞.

Let us look at some examples:
1. A number line showing rational number 1/2 and -1/2.

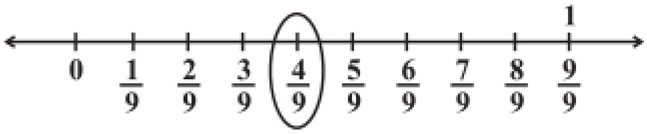


Here, ½ divides the distance between 0 and 1 into two equal parts.

2. Similarly, 1/8 can be represented by diving distance between 0 and 1 into eight equal parts.

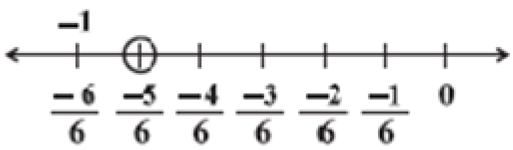


From examples (1) and (2), we can say that any rational number can be shown on the number line. For any given rational number, the denominator informs about the number of equal parts into which the first unit has been divided and numerator informs about ‘how many’ of these parts are to be considered.
For example, a rational number 4/9 means four of nine equal parts on the right of 0.

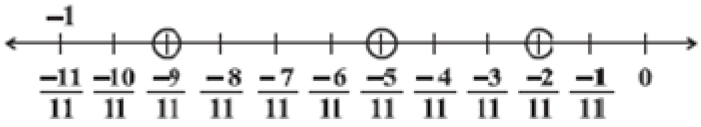


Some Examples:

***Example 1*: Represent** $\frac{-5}{6}$ on the number line.
Solution:



***Example 2*:** Represent on the number line.
Solution



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2. Rational Numbers between Two Rational Numbers:

There can be infinite rational numbers between any two given rational numbers.

***Example 1*:** How many rational numbers can exist between -2/10 and 2/10?
Solution: We can write -2/10 as -20000/100000 and 2/10 as 20000/100000, now, we can write infinite rational numbers between them as -19999/100000, -19998/100000, -19997/100000, etc.

***Example 2*:** Find ten rational numbers between $\frac{3}{5}\ and\ \frac{3}{4}$

Solution: The given numbers can be written as $\frac{3\times 20}{5\times 20}\ and\ \frac{3\times 25}{4\times 25}\ = \frac{60}{100}\ and\ \frac{75}{100}$

Hence, ten rational numbers between 3/5 and 3/4 can be
$\frac{61}{100}, \frac{62}{100}, \frac{63}{100}, \frac{64}{100}, \frac{65}{100}, \frac{66}{100}, \frac{67}{100}, \frac{68}{100}, \frac{69}{100}, \frac{70}{100}$